BTRY 4830/6830: Quantitative Genomics and Genetics Fall 2014

Homework 1 (version 3)

Assigned September 3; V2 posted September 4 (minor correction); V3 posted September 8 (minor correction); Due 11:59PM September 8

Problem 1 (Easy)

a. Consider a case where we have a system of interest, where we have defined a question that we would like to answer, and an experiment that we will use to answer this question. For the sample space for this experiment and the associated Sigma-algebra, we have many options for how we could define a probability model. Explain why we might justify assuming one specific probability model over another (many acceptable answers)?

b. Using two sentences at most, provide an intuitive explanation as to why disjoint events (that have non-zero probabilities) cannot be independent. Do not write down any formulas for this intuitive explanation.

Problem 2 (Medium)

Assume that the system we are interested in is a coin. The experiment we will consider is two flips of the coin. Note: for parts (d)-(j) use the probability model in part (c).

a. What is the sample space of this experiment?

b. What is the Sigma-algebra (containing all events) for this sample space? Which of these events is the event 'the first flip is heads'? Which of these events is the event 'the second flip is heads'?

c. Define a probability model such that the probability of a 'heads' on the first flip and the second flip is $Pr(H_{1st}) = Pr(H_{2nd}) = 0.5$, where the probability of heads on both the first and second flip is $Pr(H_{1st} \cap H_{2nd}) = 0.3$. Write out the probabilities for all possible outcomes of an experimental trial. Write down the formulas or relationships you used to calculate these probabilities as part of your answer.

d. Define the random variables X_1 that is the 'number of heads on the first flip' and X_2 that is the 'number of heads on the second flip'. For X_1 and X_2, write out the values that these random variables take for all possible outcomes of an experimental trial.

e. Are X_1 and X_2 independent? Demonstrate that your answer is correct.

f. Calculate $\mathrm{E}X_1$, $\mathrm{E}X_2$, $Var(X_1)$, $Var(X_2)$, $Cov(X_1, X_2)$, and $Corr(X_1, X_2)$. Show your work for each calculation.

g. Define a new the random variables X_3 that is 'the number of heads observed for both flips'. Write out the values that this random variable takes for all possible outcomes of an experimental trial and the probabilities of each outcome.

h. Are X_1 and X_3 independent? Demonstrate that your answer is correct.

i. Calculate $Cov(X_1, X_3)$ and show your work.

j. Your answer to (i) should be a positive number. Using no more than two sentences, provide an intuitive explanation as to why this is the case.

Problem 3 (Difficult)

Consider two random variables X_1 and X_2 that have a joint probability distribution function $Pr(X_1, X_2)$. Prove that if

$$Cov(X_1, X_2) \neq 0 \tag{1}$$

then X_1 and X_2 are NOT independent. Note that for full credit, you must demonstrate this for the general case, i.e. you will not get full credit for a specific example.