Quantitative Genomics and Genetics - Spring 2016
BTRY 4830/6830; PBSB 5201.01

Homework 1 (version 1)

Assigned February 3; Due 11:59PM February 8

Problem 1 (Easy)

Consider a coin (system) that you would like to learn about and two types of experiments: 1. One flip of the coin (Experiment 1), and Two flips of the coin (Experiment 2). Consider the case where you are going to perform TWO experimental trials for Experiment 1 and ONE experimental trial for Experiment 2.

a. Write out BOTH the sample spaces AND the Sigma Algebras for BOTH Experiments 1 and 2.

b. Using one sentence at most, explain why the sets $\{H_1, T_2\}$ describing the sample of a 'heads' on the first trial and a 'tails' on the second trial, resulting from the two trials of Experiment 1, is distinct from the set $\{HT\}$ describing the results of one trial of Experiment 2.

c. Answer the following questions by considering possible sample outcomes for Experiment 1 (two total trials). What is $\{H_1, H_2\} \cap \{H_1, T_2\}$?

c. Answer the following questions by considering possible sample outcomes for Experiment 2 (one total trial). What is $\{HH\} \cap \{HT\}$?

Problem 2 (Medium)

Consider a coin that you plan to learn about with the following experiment: 3 flips of the coin.

a. Write out the sample space of this experiment.

b. For the (appropriate) Sigma Algebra of this experiment, how many subsets will be the null / empty set? How many subsets will have one element? How many subsets will have two elements? How many subsets will have eight elements? Note that you DO NOT need to write out these sets or the entire Sigma Algebra!

c. Define a probability model such that each of the subsets of the Sigma Algebra that have ONE element each are assigned THE SAME probability. Explain why from this starting point, you can calculate the probabilities of all of the other subsets of the Sigma Algebra. Note that you DO NOT need to write out these probabilities for the other subsets of the Sigma Algebra - just explain your reasoning (which may include an example or two).

d. Define the random variables X_1 that is the 'number of heads on the first flip' and X_2 that is the 'number of heads on the second flip'. For X_1 and X_2, write out the values that these random variables take for each of the possible sample outcomes that could result from a single experimental trial.

e. What is $Pr(X_1 = 1)$? What is the $Pr(X_2 = 1)$? Explain how you arrived at these answers.

f. What is $Pr(X_2 = 1|X_1 = 1)$? Show your work by making use of the formula for conditional probability.

g. Explain why your answer to part 'f' is consistent with X_1 and X_2 being independent and why this makes sense given the probability model.

h. Define a new the random variable X_3 that is 'the number of heads observed for all three flips'. Write out the values that this random variable takes for each of the possible outcomes of a single experimental trial.

i. What is $Pr(X_3 = 3|X_1 = 1)$? Show your work by making use of the formula for conditional probability.

j. Explain why your answer to part 'i' is consistent with X_1 and X_3 NOT being independent and why this makes sense intuitively given how these random variables are defined.

Problem 3 (Difficult)

a. Consider a system and associated experiment where the sample space is the set of Natural numbers. Make use of the Axioms of Probability to show why we cannot define a probability function on the (appropriate) Sigma Algebra of this sample space that assigns an equal probability to each (non-empty set) subset of the Sigma algebra that has a single element.

b. Consider a system and associated experiment where the sample space is the subset of the Reals from zero to one (including zero and one) and the number 2 (i.e., $\Omega = [0, 1] \cup 2$). Consider a random variable X on this sample space that is the identity function. Write out the equation of the probability distribution function of X where the probability of each interval within $[0, 1]$ of the same size has an equal probability AND where the probability of the entire interval $[0, 1]$ and the number 2 is the same. Also, write out the equation for the cumulative distribution function for X for the probability distribution function just described.

   Note that in both cases, the distribution equations will not be in 'closed' form (i.e., you cannot write them out as a single equation), such that you will need to write out the equation in two pieces, part for $X \in [0, 1]$ and part for $X \in 2$. On a side note, what you are writing out a 'mixture' distribution for X, which is a mixture of continuous and discrete.