Homework 2 (version 1)

Assigned Feb. 6; Due 11:59PM February 12

Problem 1 (Easy)

a. Consider a random variable $X$ that has an associated probability density function (pdf), i.e. $X$ is a continuous random variable. What is the probability of observing a specific (point) value of $X$? Given that this is the case, how is it that we can use $X$ to model outcomes generated by a real experiment (you may use an example to illustrate your answer)? Limit your answer to at most three sentences.

b. Consider a random variable $X$ that has a normal probability distribution $X \sim N(\mu, \sigma^2)$ for which $\mu = 0$. What is the value of the cumulative density function $F_X(x)$ at $x = 0$? Provide an intuitive explanation (no formulas!) as to why you know that this is the answer in terms of the ‘shape’ of the normal distribution and how a cumulative density function works. Limit your answer to at most two sentences.

Problem 2 (Medium)

Note: DO NOT use R functions to do the calculations in your answers below or to show how you did the calculations. You may use R markdown to present your solutions but within your markdown document, show the steps used to calculate each of the numbers required for each of your answers.

For all of the parts of Problem 2, consider a coin and a two flip experiment. Assume a probability function defined on the Sigma Algebra (which you do not need to specify explicitly for this question) that assigns the following probabilities to each two flip event:

$$Pr(HH) = 0.4, Pr(HT) = 0.1, Pr(TH) = 0.1, Pr(TT) = 0.4$$

(1)

Consider a random variable $X_1$ that takes the value ‘1’ if the first flip is ‘Heads’ and the value ‘0’ if the first flip is not ‘Heads’. Also consider a random variable $X_2$ that takes the ‘1’ if the second flip is ‘Heads’ and the value ‘0’ if the second flip is not ‘Heads’.

a. Write out the joint (bivariate) distribution for random variables $X_1$ and $X_2$, i.e., your answer should be of the form $Pr(X_1 = 0, X_2 = 0) =$?, $Pr(X_1 = 1, X_2 = 0) =$?, etc..
b. Write out the marginal probability distributions of $X_1$ and $X_2$ and show how you calculated each of these probabilities, i.e., You need to show how you calculated $Pr(X_1 = 0) =$ ? , $Pr(X_1 = 1) =$ ? and similarly for $X_2$.

c. Write out the conditional distribution of $Pr(X_1|X_2 = 1)$, i.e., your answer should be of the form $Pr(X_1 = 0|X_2 = 1) =$ ?, $Pr(X_1 = 1|X_2 = 1) =$ ?. Show how you calculated each of these probabilities.

d. Demonstrate that $X_1$ and $X_2$ are not independent.

e. Calculate the expected values of $X_1$ and $X_2$. Show how you calculated each of these expectations.

f. Calculate the variances of $X_1$ and $X_2$. Show how you calculated each of these variances.

g. Calculate $Cov(X_1, X_2)$. Show how you calculated the covariance.

h. Explain why you knew your answer to [g] would not be zero given your answer to part [d]?

i. Was your answer to part [g] positive or negative? What does this imply about the $Pr(X_1 = 1, X_2 = 1)$ versus the probability of $Pr(X_1 = 1, X_2 = 0)$ and $Pr(X_1 = 0, X_2 = 1)$ and similarly for the $Pr(X_1 = 0, X_2 = 0)$ versus the probability of $Pr(X_1 = 1, X_2 = 0)$ and $Pr(X_1 = 0, X_2 = 1)$.

j. Calculate $Corr(X_3, X_4)$. Show how you calculated the correlation.

**Problem 3 (Difficult)**

Prove that it is not possible to define a probability function for three random variables $X_1$, $X_2$, $X_3$ such that $corr(X_1, X_2) = corr(X_1, X_3) = corr(X_2, X_3) = -1$. **Hint:** use the variance of a sum of random variables.